Convolution continued from last day.

Method 3 (mathematical)

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= \sum_{k=-6}^{3} x[k]h[n-k]$$

$$\begin{aligned} &= x[-6]h[n+6] \\ &+ x[-4]h[n+4] \\ &+ x[-2]h[n+2] \\ &+ x[0]h[n] \\ &+ x[3]h[n-3] \end{aligned}$$

$$\begin{aligned} &= -h[n+6] \\ &+ h[n+4] \\ &+ h[n+2] \\ &+ 2h[n] \\ &- h[n-3] \end{aligned}$$

Note: if $n \leq -7$, $y[n]=0$

$n \geq 6$, $y[n]=0$

$\therefore$ $y[-6] = -h[0] + h[-2] + \ldots$

Method 4 (use z transform) easist

Method 5 (Use MatLab)

1.4 Properties of LTI system

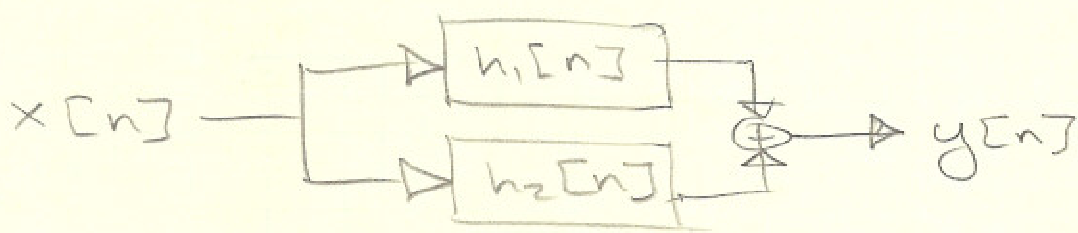
A) Commutative property

$$y[n] = x[n] * h[n]$$
$$= h[n] * x[n]$$

B) Distributive property.

$$x[n] * (h_1[n] + h_2[z])$$
$$= x[n] * h_1[n] + x[n] * h_2[n]$$

c) Parrellel LTI

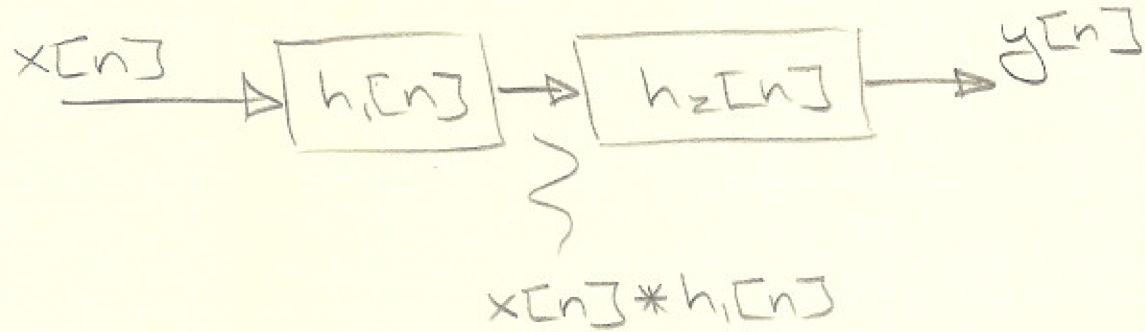


$$y[n] = x[n] * h_1[n]$$
$$+ x[n] * h_2[n]$$

$$= x[n] * (h_1[n] + h_2[n])$$

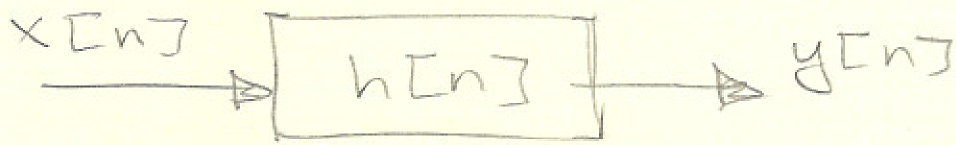
$$\therefore \ h[n] = h_1[n] + h_2[n]$$

D) Cascade LTI



$$y[n] = x[n] * h_1[n] * h_2[n]$$

Hence it is the same as



Where:

$$h[n] = h_1[n] * h_2[n]$$

EX: Consider a system w impulse responce

$$h[n] = U[n] - U[n-N]$$

Find the responce of the i/p

$$x[n] = a^n U[n]$$

SOL:

$$x[n] = a^n U[n] \qquad \boxed{|a| < 1}$$

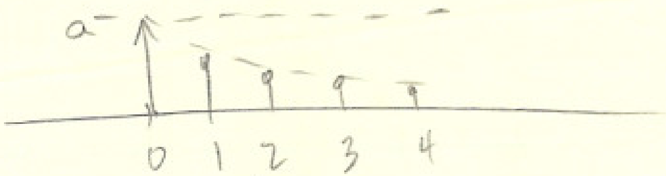
$$= \begin{cases} a^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$h[n] = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & \text{elsewhere} \end{cases}$$

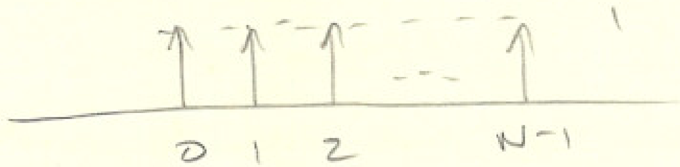
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

Note We try to avoid shifting the exponential sequence.

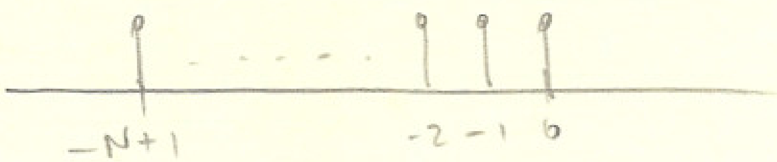
$x[k] =$



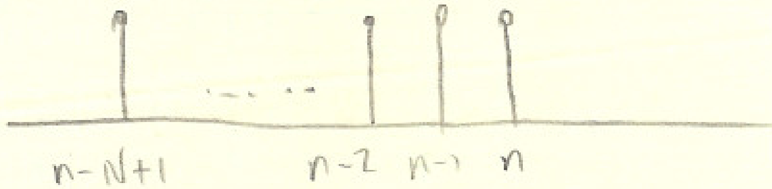
$h[n]$



$h[-k]$



$h[n-k]$



$\therefore$ if $n<0$, $y[n]=0$

if $n \geq 0$ & $n-N+1 \leq 0$
or $0 \leq n \leq N-1$

if $n \geq N-1$

$$y[n] = \sum_{k=0}^{n} a^k$$

$$y[n] = \sum_{k=n-N+1}^{n} a^k$$

Note: $$\boxed{\sum_{k=N_1}^{N_2} \alpha^k = \frac{\alpha^{N_1} - \alpha^{N_2+1}}{1-\alpha}}$$

1.5 correlation of degree $\boxed{\text{pg } 118}$

The correlation is a messure of the degree to which two sequence are similiar. The correlation of seq. $x[n]$

w $y[n]$ is a seq $r_{xy}[l]$ defined as:

$$r_{xy}[l] = \sum_{n=-\infty}^{\infty} x[n]\, y[n-l]$$

$$\boxed{l \in (-\infty, \infty)}$$

$$= \sum_{n=-\infty}^{\infty} x[n]\, y[-(l-n)]$$

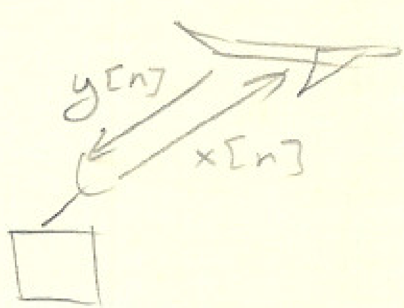
$$= x[l] * y[-l]$$

Hence, correlation can be determined by convolution. If $x[n] = y[n]$ then it is called autocorrelation:

$$r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n]\, x[n-l]$$

Application: The correlation can be used to identify and localise the target in radar signal processing.

ie - the depth of the ocean floor can be measured by sending a signal from a transmitter and then by correlating this signal with the recieved signal.



attinuation factor

$$y[n] = \alpha\, x[n-k] + w[n]$$

round trip delay.

noise

If no target is detected the recieved signal is all noise.

Thus by correlation, distance of the target can be messured by messuring the time delay K. Auto correlation seq. attains it's max value at zero shift which indicates that the signal matches perfectly at zero lag.